**D3.3 (WP3): Scalable SD Erlang Reliability Model**

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**Purpose:** Implement and validate a scalable reliability model for SD Erlang.

**Results:** The main results of this deliverable are as follows.

- We have implemented a node recovery mechanisms for SD Erlang `s_groups`.
- We have minimised the probability `s_groups` with the same name occurring, and ensured correct node behaviour in case duplicate names occur.
- We have provided an `s_group` operational semantics together with a QuickCheck correctness check of the `s_group` module functions.

**Conclusion:** We have implemented a node recovery mechanism, and both specified and validated correctness of `s_group` SD Erlang functions.

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Scalable SD Erlang Reliability Model

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1 Executive Summary

This deliverable presents the work we have conducted to support SD Erlang reliability and to formally specify the semantics of s_groups. We have considered the following aspects of SD Erlang reliability: node recovery after failures and s_group name uniqueness.

The first aspect is restarting nodes in their s_groups after failures (Section 3). We have considered a number of approaches to implement node restart, and have implemented one of them. The mechanism is optional, and records s_group state information for nodes which are started with the -rsconfig flag.

The second aspect considers s_groups with the same name (Section 4). In SD Erlang we do not guarantee s_group name uniqueness; however, we guarantee a high probability of s_group name uniqueness when s_groups are started dynamically – for that we have modified function s_group:new_s_group. When s_groups are started at launch the configuration file is checked to minimise occurrence of s_groups with the same name. We have also ensured correct behaviour of s_groups and nodes in case duplicate s_group names occur.

We introduce an operational semantics for SD Erlang s_groups that defines an abstract state and presents transition of fifteen SD Erlang functions (Section 5). We have conducted a correctness check of the implemented functions using QuickCheck [AHJW06] testing tool (Section 6).
2 Introduction

The objectives of Task 3.3 are to “implement and validate a scalable reliability model for SD Erlang by minimising supervision interconnections”. The lead participant is the University of Glasgow.

SD Erlang reliability includes a wide spectrum of issues, such as node recovery after failures, correctness of Erlang node behaviour in s_groups, and correctness of implemented functions. We start with a discussion on node recovery after failures in Section 3. Here, we cover a wide range of possible node recovering mechanisms, and implement one of them in SD Erlang. Then we discuss s_groups with the same name in Section 4. We started checking the correctness of the functions in deliverable D3.2: Scalable SD Erlang Computation Model by conducting an extensive unit testing [REL13a]. In the current deliverable we introduce SD Erlang semantics. We describe fifteen SD Erlang functions in mathematical terms using state transition semantics in Section 5. Then we compare results of mathematical calculations and of the real implementation using QuickCheck [AHJW06] testing tool in Section 6.

Partner Contributions to WP3. We had a series of teleconferences with University of Kent and Erlang Solutions. The University of Kent contributed to the development of SD Erlang semantics and conducted the QuickCheck experiments. The Ericsson and ESL teams contributed to identifying the reliability issues and requirements to the SD Erlang. Enrique Fernandez from ESL visited the University of Glasgow to identify the interaction between the Wombat deployment and recovery tool developed in WP6 [REL13b] and reliable SD Erlang developed in WP3.

3 Restarting a Failed Node

The mechanism for restarting a failed node consists of three main components: collecting s_group state information of the node, detecting the node failure, and restarting the node using the s_group state information. In this section we first discuss different strategies to collect node s_group information (Section 3.1). Then we consider strategies that would be useful in the benchmarks we use in the RELEASE project (Section 3.2), and finally we present the strategy that we have implemented in SD Erlang (Section 3.3).

3.1 Recovering S_group State

A node may join or leave s_groups, so we need to record s_group membership information to correctly restart a failed node.

The information collection mechanism can be implemented in a number of ways. To explore different alternatives we use system state estimation, $\xi$, that analyses where state information is collected, with whom it is shared, who initiates the exchange and how often it happens [Rot94]. More formally,

$$\xi \in \{\text{Centralised, Decentralised, Hybrid}\} \times \{\text{Complete, Partial, Variable}\} \times \{\text{Voluntary, Involuntary, Composite}\} \times \{\text{Periodic, Aperiodic, Combination}\}$$

Below we discuss the meaning of the parameters and examples of how a particular property can be implemented to collect s_group information.

The first set (Centralised, Decentralised, Hybrid) defines where s_group state information is collected. In case of a centralised scheme a dedicated node or a process may collect s_group state information from the whole system. A decentralised scheme implies that each node is responsible for storing its own
s_group state information, for example, in a .config file. A hybrid scheme combines both centralised and decentralised schemes, e.g. an s_group leader node collects s_group state information from the members of its s_group in a centralised manner, and then s_group leader nodes exchange the state information between each other in a decentralised manner.

The second set (Complete, Partial, Variable) defines with whom s_group state information is shared. A complete scheme implies that every process that collects s_group state information has information about all nodes of the system. In a partial scheme an information collecting process has only information about the nodes it collects information from. A variable scheme means that some processes have partial information and some processes have full information. The scheme is suitable for the cases when information is collected in a hierarchical manner, e.g. processes of Level1 collect information from the nodes and pass it to processes of Level2. Thus, Level1 processes have up-to-date partial information; whereas Level2 processes have information about all nodes in the system, but this information may be out-of-date.

The third set (Voluntary, Involuntary, Composite) defines who initiates the information exchange. In a voluntary scheme nodes initiate sending s_group state information to the collecting process themselves. In an involuntary scheme nodes send s_group state information as a response on a request to provide the information. In a composite scheme some information is send voluntary and some is sent involuntary. For example, nodes send their s_group information to Level1 processes voluntary, whereas Level1 processes send information to Level2 processes involuntary.

The fourth set (Periodic, Aperiodic, Combination) defines how often the information exchange happens. The periodic and aperiodic schemes are self explanatory, for example, information is exchanged every 30 sec or every time the node s_group state changes. In a composite scheme nodes may send s_group state information to Level1 processes aperiodically, and Level1 processes send information to Level2 processes periodically.

The system state estimation analysis shows that there are many ways that the s_group information collecting algorithm can be implemented. Each algorithm has target applications and some of these algorithms are more complicated than others. To identify the most suitable type of the node restarting algorithm for SD Erlang we analyse exemplars we work with in the RELEASE project [BCC+12] (Section 3.2).

3.2 Exemplars

In this section we consider four exemplars we currently use in the RELEASE project to evaluate distributed Erlang and SD Erlang, i.e. Sim-Diasca, Riak, Orbit, and Mandelbrot set. The exemplars are representatives of typical distributed Erlang applications. All benchmarks are open source. The benchmark details and their possible implementations in SD Erlang are discussed in deliverable D3.1: Scalable Reliable SD Erlang Design [REL12]. Therefore, in this section for every benchmark we only discuss principles of grouping nodes in s_groups and nodes dynamic membership of s_groups.

**Sim-Diasca** is a simulation engine developed in EDF [EDF10]. When new processes are spawned the aim is to place often communicating processes close to each other. Thus, nodes can be grouped on the basis of locality. Currently, the engine executes on a fixed number of nodes, and s_group configuration will not change much during the program execution. Therefore, nodes can be grouped in s_groups either at the node launch or dynamically before the actual simulation has started.

**Riak** is a database management system developed in Basho [Klo10]. The Riak’s aim is to provide access to the stored data even when some nodes have failed. Nodes can be grouped in s_groups on the basis of Riak preference lists, i.e. a node forms an s_group with the nodes on which its replica data is kept. Thus, each node will have its own s_group. When a node fails its s_group information will be
redistributed and the s_group will be deleted. When a node gets back again it will join s_groups of the neighbouring nodes and create its own s_group. The node will know its neighbours as they will support its restart; from the neighbours the node will know which nodes to add to its own s_group and which s_groups to join. Thus, Riak nodes will join and leave s_groups dynamically, and essential s_group information will be provided by peer nodes.

**Orbit** is a computation oriented, distributed hash table benchmark. A hash table that collects results of calculations is partitioned between the nodes. The aim of the application is to explore a given set of values using a set of generators and fill the table. The s_group configuration is static. Orbit can be implemented in failure resistant or failure non-resistant manner. In a non-resistant implementation the Erlang nodes keep only their own partitions of the table; so, when a node fails its part of the table is lost, and Orbit should be restarted. In the failure resistant implementation the partitions are replicated on the neighbouring nodes; thus, when a node fails we may want to restart it in its s_groups.

**Mandelbrot Set** is a computation oriented benchmark. The benchmark uses escape time algorithm to determine a colour of every pixel on a plot of a given size, i.e. the more iterations it is required for a value to reach an escape point the darker the pixel’s colour. The size of the computation depends on the size of the image, the number of iterations to reach an escape condition, and complexity of the escape condition. The s_group configuration is dynamic, the more resources the computation requires the larger number of nodes are involved in the computation. The nodes can be grouped on the basis of locality. When a node fails its computation will be re-spawned to the remaining nodes. If the system requires more resources a new node will be started. Thus, a failure mechanism is not needed.

The exemplars show that applications require different node restarting algorithms if any, i.e. Riak uses information form neighbouring nodes and Mandelbrot set does not require node restarting. In Sim-Diasca and Orbit the nodes can be grouped in s_groups either at launch or dynamically before the programs actually start. In case the s_groups are started at the node launch and the s_groups never change the failed nodes can be restarted using the initial configuration. In case s_groups are started dynamically we have implemented a simple optional node restarting algorithm. The details of the algorithm are presented in Section 3.3.

### 3.3 Implemented Approach

Applications require different node recovery mechanisms as discussed in Sections 3.1 and 3.2. Therefore, we have implemented a simple mechanism that suits our benchmarks and can be used by setting the -rsconfig flag at the launch of Erlang nodes. From (1) the approach has the following system state estimation, $\xi$, parameters:

$$\xi \equiv (\text{Decentralised}, \text{Partial}, \text{Voluntary}, \text{Aperiodic})$$ (2)

i.e. each node keeps its own s_group information in a configuration file that is updated by the node every time the node s_group configuration changes. The details of the algorithm are as follows. When a node is started and the -rsconfig flag is set a new configuration file named NodeName.config is created. Every time s_group information is renewed s_group process spawns a process to update the configuration file. The configuration is only updated if the new state information timestamp is greater than the last update. The configuration file is kept in the same directory as the running node.

Consistent with the SD Erlang design principles [REL12, Section 3], reliable s_groups leave distributed Erlang recovery mechanisms unchanged. In distributed Erlang a node failure can be detected in several ways, e.g. using a supervision behaviour, a heartbeat monitoring by setting the -heart flag, or net_adm:world_list/1,2 and erlang:monitor_node/2,3 functions. To restart a failed node a
script file or `slave:start/3` function can be used. However, when using `slave:start/3` function one should remember that slave nodes automatically terminate when the master process that started the nodes terminates.

4 S_groups with the Same Name

It is very difficult to guarantee name uniqueness in a distributed system. We have analysed a number of different approaches, and here is a summary of our analysis.

- Centralised schemes are not scalable and hence are not suitable.

- A flat decentralised scheme either causes a sharp increase of message exchange when an s_group is added or requires non-friendly user names, such as Universally Unique IDentifiers (UUIDs) [LMS05].

- A hierarchical decentralised scheme provides a straightforward solution for name uniqueness but this will require s_groups to be structured in a hierarchical manner which is not suitable for SD Erlang.

- A hybrid scheme also does not seem suitable, i.e. centralised within an s_group and decentralised between s_groups. For example, a leader s_group node keeps information about all s_groups its nodes belong to. Then when a node attempts to create a new s_group only s_group leader nodes exchange messages between each other to check whether an s_group with the same name already exists. However, when the number of s_groups is compatible with the number of nodes the scheme has the same issues as a flat decentralised scheme.

- A leader node approach, i.e. a full s_group name consists of an s_group name and the leader node name, for example, `group1@node1@glasgow.ac.uk`. However, the scheme becomes very complicated when leader nodes together with other s_group nodes fail and we need to pass the s_group leadership and identify s_groups to which the restarted nodes should be added.

Therefore, instead of guaranteeing s_group name uniqueness we guarantee a high probability of name uniqueness and explore node behaviour in cases when s_group names are not unique. For that we modify `s_group:new_s_group` function, i.e. from

\[
s_{\text{group}}:\text{new}_{\text{s_group}}(\text{SGroupName, [Node]} \rightarrow \{\text{SGroupName, [Node]}
\]

to

\[
s_{\text{group}}:\text{new}_{\text{s_group}}([\text{Node}] \rightarrow \{\text{SGroupName, [Node]}
\]

S_group names are generated by `s_group:new_s_group/1` using UUIDs. In SD Erlang we use `uuid` module. In the future this UUID generator may be changed to decrease a probability of generating two identical UUIDs. A discussion on UUID randomness can be found in [IOS10].

In this section we discuss the cases when there are s_groups with the same name and focus on the following two issues.

1. Ensuring that when two nodes from s_groups with the same name connect they behave in a predictable manner.

2. Preventing nodes from joining an s_group when it may cause a confusion, e.g. a node cannot belong to a number of s_groups with the same name.

A mathematical description of interconnections between s_groups with the same name is presented below. Here, \( x, y \) are nodes; \( A, A', B \) are s_groups; and \( A, A' \) s_groups have the same s_group name.

- **Disjoint s_groups** contain no nodes which belong to both s_groups that share a name (Figure 1(a)), i.e. \( \exists x (x \in A, x \in A') \land \forall x, y \exists B (x \in A, y \in A', [x, y] \in B) \).
(a) Disjoint s_groups
(b) Directly overlapping s_groups
(c) Indirectly overlapping s_groups

Figure 1: Interconnections between S_groups with the Same Name

- **Directly overlapping s_groups** contain nodes that belong to multiple s_groups that have the same name (Figure 1(b)), i.e. \( \exists x \ (x \in A, x \in A') \).

- **Indirectly overlapping s_groups** have no nodes that belong to more than one s_group with a duplicate name but some nodes belong to an s_group with a different name and through it the nodes share a namespace (Figure 1(c)), e.g.
  \[ \not\exists x \ (x \in A, x \in A') \land \exists x, y, B \ (x \in A, y \in A', [x, y] \in B) \]

When we create an s_group or add nodes to an existing s_group we should ensure that no nodes in that s_group belong to different s_groups with the same name. A node may become a member of multiple s_groups that have the same name either when it is started, i.e. at the node launch using s_group configuration, or when nodes are added to existing s_groups, i.e. dynamically using s_group:new_s_group/2 and s_group:add_nodes/2 functions. Below we discuss s_group interaction for the case when nodes join s_groups at launch (Section 4.1) and dynamically (Section 4.2).

### 4.1 Joining an S_group at Launch

To join an s_group at the node launch the -config flag together with a configuration file are used. The configuration file contains information about prospective s_groups. For example, to start a node using s_group.config.config file the following command is called:

```
$ erl node1@glasgow.ac.uk -config s_group.config
```

An example of configuration file s_group.config.config is presented in Listing 1.

**Listing 1: An example of s_group.config.config file**

```erlang
%%kernel, [ [s_groups, [ [group1, normal, [‘node1@glasgow.ac.uk’, ‘node2@glasgow.ac.uk’,
  ‘node3@glasgow.ac.uk’, ‘node4@glasgow.ac.uk’]],
  [group2, normal, [‘node3@glasgow.ac.uk’, ‘node5@glasgow.ac.uk’,
  ‘node6@glasgow.ac.uk’]],
  [group3, normal, [‘node4@glasgow.ac.uk’, ‘node7@glasgow.ac.uk’,
  ‘node8@glasgow.ac.uk’]]]]
```

Thus, at the node launch we can check the s_group configuration and eliminate undesirable behaviour. The configuration file may contain information either only about own s_groups or about own and remote s_groups. In case the configuration file contains descriptions of multiple s_groups with the same name one of the following actions can be done.

- Merge s_groups that have the same name.
- Consider only the first encounter of the s_group name and ignore the rest.
(a) Disjoint s_groups  
(b) Directly overlapping s_groups  
(c) Indirectly overlapping s_groups

Figure 2: S_group Interconnections when the S_groups are Joined at Launch

- Discard s_groups whose names occur more than once in the configuration file.
- Report the error in the configuration file and start the node as a free node.
- Report the error and terminate the node using `init:stop()` function.

From the above alternatives we have implemented the following. In case all s_groups that have the same name are identical we create a single s_group; otherwise we check whether any of these s_groups contain the current node. In case the s_groups that have the same name are remote we keep a record of no s_group with that name. In case the current node is a member of at least one s_group an error is returned and the node terminates.

The reason we have decided to terminate the node in case the configuration is wrong is to inform the programmer that the configuration for a particular node is dubious and should be specified to avoid wrong connections and incorrect information sharing. As for the information about the remote nodes, currently, we do not renew s_group information but trust that it is correct, and if the information is ambiguous from the beginning it is better to discard it to eliminate errors.

The cases in which the configuration file contains information only about one s_group that have the same with other s_groups in the system are discussed below. We consider the following types of overlapping: disjoint, direct and indirect overlapping (Section 4).

**Disjoint S_groups.** In case the node has a record of only one s_group with a particular name the node is started and no s_group information is modified. For example, we start four nodes: nodes N1 and N2 are started with configuration from Listing 2, and nodes N3 and N4 are started with configuration from Listing 3. Nodes from different disjoint s_groups communicate normally and do not share the namespace (Figure 2(a)).

**Listing 2: Configuration 1 for Disjoint S_groups**

```erlang
[{kernel, [{s_groups, [{group1, normal, ['node1@glasgow.ac.uk', 'node2@glasgow.ac.uk']}]}}].
```

**Listing 3: Configuration 2 for Disjoint S_groups**

```erlang
[{kernel, [{s_groups, [{group1, normal, ['node3@glasgow.ac.uk', 'node4@glasgow.ac.uk']}]}}].
```
Directly Overlapping S_groups may occur when nodes have inconsistent information about S_groups. For example, we start four nodes: nodes N1 and N2 are started with configuration from Listing 4, and nodes N3 and N4 are started with configuration from Listing 5. As nodes N2 and N3 have different information about group1 the nodes do not synchronise and have a synchronisation error, i.e. nodes N1 and N2 belong to one S_group and nodes N3 and N4 belong to another S_group that have the same name group1. The S_groups do not share connections and namespace (Figure 2(b)).

```
Listing 4: Configuration 1 for Directly Overlapping S_groups

[[kernel, [[[s_groups,
    [[group1, normal, ['node1@glasgow.ac.uk', 'node2@glasgow.ac.uk',
    'node3@glasgow.ac.uk']]])]]]]] '

Listing 5: Configuration 2 for Directly Overlapping S_groups

[[kernel, [[[s_groups,
    [[group1, normal, ['node2@glasgow.ac.uk', 'node3@glasgow.ac.uk',
    'node4@glasgow.ac.uk', 'node5@glasgow.ac.uk']]]]]
```

Indirectly Overlapping S_groups may occur when nodes have partial information about remote S_groups. For example, we start seven nodes: nodes N1, N2, N3 are started using configuration from Listing 6, and nodes N4, N5, N6, N7 are started using configuration from Listing 7. Independently of the order we start the nodes they do not share connections and name spaces (Figure 2(c)).

```
Listing 6: Configuration 1 for Indirectly Overlapping S_groups

[[kernel, [[[s_groups,
    [[group1, normal, ['node1@glasgow.ac.uk', 'node2@glasgow.ac.uk',
    'node3@glasgow.ac.uk']],
    {group2, normal, ['node3@glasgow.ac.uk', 'node4@glasgow.ac.uk',
    'node5@glasgow.ac.uk', 'node6@glasgow.ac.uk']}]])])]]

Listing 7: Configuration 2 for Indirectly Overlapping S_groups

[[kernel, [[[s_groups,
    [[group1, normal, ['node5@glasgow.ac.uk', 'node6@glasgow.ac.uk',
    'node7@glasgow.ac.uk', 'node5@glasgow.ac.uk']],
    {group2, normal, ['node3@glasgow.ac.uk', 'node4@glasgow.ac.uk',
    'node5@glasgow.ac.uk', 'node6@glasgow.ac.uk']}]])])]]
```

4.2 Joining an S_group Dynamically

A node can join an S_group dynamically using S_group:new_S_group/1 or S_group:add_nodes/2 functions defined in Sections 4, 5.2.1, and in [REL12]. In both cases we distinguish two types of nodes: a node that initiates the action and nodes that join the S_group. Additionally, S_group:add_nodes/2 function has the third type of nodes, i.e. nodes which are already members of the S_group.

Disjoint S_groups may occur when S_groups with the same name are started independently from each other. When nodes from disjoint S_groups connect they share neither connections nor namespaces.
Directly Overlapping S_groups. To prevent directly overlapping s_groups the initiating node first checks whether the new s_group’s name is identical to an s_group the node already belongs to. If it is then a new s_group name is generated and the checking is repeated. When a node receives a request to join an s_group it also first checks whether it already belongs to an s_group with the same name. Only nodes that return a negative response are added to the s_group.

Indirectly Overlapping S_groups may occur as a result of using s_group:new_s_group/1 or s_group:add_nodes/2 functions. When nodes from indirectly overlapping s_groups connect they do not share connections and namespaces.

4.3 Summary.

In this section we have discussed dynamic and at launch s_group creation, and measures we have taken in SD Erlang to minimise the probability of presence of s_groups that have the same name in the system (Sections 4 and 4.1). For each type of s_group creation we have discussed three ways of interconnections between s_groups that have the same name, i.e. disjoint s_groups, directly overlapping s_groups, and indirectly overlapping s_groups (Sections 4.1 and 4.2). Here, have covered the cases when the interconnections may occur, and how nodes and the system behave.

We recommend using at launch state configuration only in cases when nodes need to know information about other s_groups in the system from the start, and s_group configuration is not likely to change. It is up to the programmer to ensure that at launch s_group configuration is correct, and no s_groups have identical s_group names, e.g. starting the whole system with a single configuration file to avoid copy-paste errors. Otherwise, dynamic s_group creation provides a higher probability of unique s_group names. To avoid s_group name clash we also advise using either only static or only dynamic s_group creation.

5 S_group Operational Semantics

As requested at the 1st year review we present a small-step transition semantics for the s_group operations provided by SD Erlang. For simplicity the current semantics ignores failures. We start by defining an abstract state of SD Erlang systems (Section 5.1), and then define each operation as a transition between states (Section 5.2).

5.1 SD Erlang State

The state of an SD Erlang system, and associated abstract syntax variables, are defined in Figure 3. The state of a system is modelled as a four tuple comprising a set of s_groups, a set of free_groups, a set of free_hidden_groups, and a set of nodes. Each type of groups is associated with nodes and has a namespace. An s_group additionally has a name, whereas a free_hidden_group consists of only one node, i.e. a hidden node simultaneously acts as a node and as a group, because as a group it has a namespace but does not share it with any other node. Free normal and hidden groups have no names, and are uniquely defined by the nodes associated with them. Therefore, group names, gr_names, are either NoGroup or a set of s_group_names. A namespace is a set of name and process id, pid, pairs and is replicated on all nodes of the associated group.

A node has the following parameters: node_id identifier, node_type that can be either hidden or normal, connections, and group_names, i.e. names of groups the node belongs to. The node can belong to either a list of s_groups or one of the free groups. The type of the free group is defined by the node type. Connections are a set of node_ids.
\[(grs, fgs, fhs, nds) \in \{\text{state}\} \equiv \{(\{s\text{-group}\}, \{\text{free-group}\}, \{\text{free_hidden_group}\}, \{\text{node}\})\}\]
\[gr \in grs \equiv \{s\text{-group}\} \equiv \{(s\text{-group\_name}, \{\text{node\_id}\}, \text{namespace})\}\]
\[fg \in fgs \equiv \{\text{free-group}\} \equiv \{(\{\text{node\_id}\}, \text{namespace})\}\]
\[fh \in fhs \equiv \{\text{free_hidden_group}\} \equiv \{(\{\text{node\_id}\}, \text{namespace})\}\]
\[nd \in nds \equiv \{\text{node}\} \equiv \{(\text{node\_id}, \text{node\_type}, \text{connections}, gr\text{-names})\}\]
\[gs \in \{gr\text{-names}\} \equiv \{\text{NoGroup}, \{s\text{-group\_name}\}\}\]
\[ns \in \{\text{namespace}\} \equiv \{(\{\text{name}\}, \text{pid})\}\]
\[cs \in \{\text{connections}\} \equiv \{\{\text{node\_id}\}\}\]
\[nt \in \{\text{node\_type}\} \equiv \{\text{Normal, Hidden}\}\]
\[s \in \{\text{NoGroup, s-group\_name}\}\]
\[n \in \{\text{name}\}\]
\[p \in \{\text{pid}\}\]
\[ni \in \{\text{node\_id}\}\]
\[nis \in \{(\{\text{node\_id}\}\}\]
\[m \in \{\text{message}\}\]

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{SD Erlang State}
\end{figure}

**SD Erlang State Property.** Every node in an SD Erlang state is a member of one of the three classes of groups: \text{s\text{-group}}, \text{free-group}, or \text{free_hidden_group}. The three classes of groups partition the set of nodes. That is, for any state \((grs, fgs, fhs, nds)\) \(\Pi_{\text{node\_id}grs, \Pi_{\text{node\_id}fgs, \Pi_{\text{node\_id}fh}}\) is a partition of \(\Pi_{\text{node\_id}nds}\) where \(\Pi_{\text{node\_id}}\) is projection onto the \text{node\_id} attribute, or set of attributes, of the tuples.

**Assumptions.** We make the following two assumptions to simplify the state transitions.

1. No two \text{s\text{-group}s} have the same name, that is all \text{s\text{-group\_names}} are unique. We discuss \text{s\text{-group\_name} uniqueness} in Section 4.

2. All \text{node\_ids} identify some node. More formally, for all \text{node\_ids} occurring in some state \((grs, fgs, fhs, nds)\), there exists some node in \text{nds} with that \text{node\_id}.

As we continue the work on the SD Erlang semantics we plan to relax these assumptions.

### 5.2 Transitions

The transitions we present in Section 5.2.1 have the following form:

\[(\text{state}, \text{command}, \text{ni}) \rightarrow (\text{state'}, \text{value})\]

meaning that executing command on node \text{ni} in \text{state} returns \text{value} and transitions to \text{state'}. The transitions use a number of auxiliary functions defined in Section 5.2.2.

In the following \(\oplus\) denotes disjoint set union. By \(xxs \equiv \bigoplus\{xx | ...\}\) we mean that elements from all generated \(xx\) sets are accumulated in one \(xxs\) set.
5.2.1 SD Erlang Functions

In this section we present the transitions of fifteen SD Erlang functions. The following nine functions change their state after the transition: `register_name/3`, `re_register_name/3`, `unregister_name/2`, `whereis_name/3`, `send/2`, `new_s_group/2`, `delete_s_group/1`, `add_nodes/2`, `remove_nodes/2`. Whereas the other six functions only return some state information but do not change the state after the transition: `send/3`, `whereas_name/2`, `registered_names/1`, `own_nodes/0`, `own_nodes/1`, `own_s_groups/0`.

The implementation and description of the functions are discussed in deliverable D3.2: Scalable SD Erlang Computational Model.

`register_name`. When registering name `n` for pid `p` in `s_group` `s` the pair `(n, p)` is added to the namespace `ns` of the `s_group` only if neither `n` nor `p` appears in the namespace, and node `ni` is a member of `s_group` `s`.

\[
\begin{align*}
((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{register}\_\text{name}(s, n, p), ni) \\
\rightarrow ((\{\{s, \{ni\} \cup \text{nis}, \{n, p\} \cup \text{ns}\}\} \cup \text{grs}', \text{fgs}, \text{fhs}, \text{nds}), \text{True}) & \quad \text{If } (n, \_ \in ns \land (\_, p) \notin ns \\
\rightarrow ((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{False}) & \quad \text{Otherwise}
\end{align*}
\]

where
\[
\{\{s, \{ni\} \cup \text{nis}, \text{ns}\}\} \cup \text{grs}' \equiv \text{grs}
\]

`re_register_name`. When re-registering name `n` for different pid `p` in `s_group` `s` the name is re-registered only if the pid is not already registered in namespace , and node `ni` is a member of `s_group` `s`.

\[
\begin{align*}
((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{re}\_\text{register}\_\text{name}(s, n, p), ni) \\
\rightarrow ((\{\{s, \{ni\} \cup \text{nis}, \text{ns}' \cup \{n, p\}\}\} \cup \text{grs}', \text{fgs}, \text{fhs}, \text{nds}), \text{True}) & \quad \text{If } (\_, p) \notin ns \\
\rightarrow ((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{False}) & \quad \text{Otherwise}
\end{align*}
\]

where
\[
\{\{s, \{ni\} \cup \text{nis}, \text{ns}\}\} \cup \text{grs}' \equiv \text{grs}
\]

\[
\text{ns}' \equiv \text{ns} - \{(n, \_\}\}
\]

`unregister_name`. To unregister name `n` from namespace `ns` of `s_group` `s` node `ni` should belong to `s_group` `s`.

\[
\begin{align*}
((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{unregister}\_\text{name}(s, n), ni) \\
\rightarrow ((\{\{s, \{ni\} \cup \text{nis}, \text{ns} - \{(n, p)\}\}\} \cup \text{grs}', \text{fgs}, \text{fhs}, \text{nds}), \text{True}) & \quad \text{If } (n, \_ \in ns \\
\rightarrow ((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{True}) & \quad \text{Otherwise}
\end{align*}
\]

where
\[
\{\{s, \{ni\} \cup \text{nis}, \text{ns}\}\} \cup \text{grs}' \equiv \text{grs}
\]

`registered_names` function returns a list of names registered in `s_group` `s` if node `ni` belongs the `s_group`, empty list otherwise. `IsSGroupSNode/3` and `OutputNs/2` are auxiliary functions defined in
Section 5.2.2.

\[
((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{registered\_names}(s), ni) \\
\rightarrow ((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), nss) \quad \text{If IsSGroupSNode}(ni, s, \text{grs}) \\
\rightarrow ((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \{\}) \quad \text{Otherwise}
\]

where

\[
\{(s, \{ni\} \oplus \text{nis}, ns)\} \oplus \text{grs}' \equiv \text{grs} \\
\text{nss} \equiv \text{OutputNs}(s, ns)
\]

**whereis\_name/2**  
function returns pid \(p\) registered as name \(n\) in s\_group \(s\) if node \(ni\) belongs to the s\_group, undefined otherwise.

\[
((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{whereis}\_name(s, n), ni) \\
\rightarrow ((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), p) \quad \text{If IsSGroupSNode}(ni, s, \text{grs}) \\
\rightarrow ((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{undefined}) \quad \text{Otherwise}
\]

where

\[
\{(s, \{ni\} \oplus \text{nis}, ns)\} \oplus \text{grs}' \equiv \text{grs} \\
p \equiv \text{FindName}(ni, s, n, \text{grs}, \text{fgs}, \text{fhs}, \text{nds})
\]

**whereis\_name/3.**  
Depending on the type of nodes \(ni\) and \(ni'\) a search of name \(n\) from group \(s\) on a remote node \(ni'\) may lead to establishing new connections between nodes \(ni\), \(ni'\), and the node on which process \(p\) actually resides.

\[
((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{whereis}\_name(ni', s, n), ni) \\
\rightarrow ((\text{grs}, \text{fgs}', \text{fhs}, \text{nds'}), p) \\
\text{where}
\]

\[
(\text{fgs'}, \text{nds'}) \equiv \text{AddConnections}(ni, ni', \text{grs}, \text{fgs}, \text{fhs}, \text{nds}) \\
p \equiv \text{FindName}(ni', s, n, \text{grs}, \text{fgs}', \text{fhs}, \text{nds'})
\]

**send.**  
In SD Erlang we have implemented two send functions, i.e. send/3 and send/4. Function send/3 is defined in Erlang as a where\_is/2 to locate pid \(p\) of process name \(n\) from s\_group \(s\), followed by send/2 to send message \(m\) to the pid (Listing 8).

**Listing 8: send/3 Function in Erlang-like Syntax**

```erlang
send(s, n, m) ->
    p = whereis\_name(s, n),
    send(p, m).
```

Function send/4 is defined in Erlang as a where\_is/3 to locate pid \(p\) of process name \(n\) from s\_group \(s\) and node \(ni'\), followed by send/2 to send message \(m\) to the pid (Listing 9).

**Listing 9: send/4 Function in Erlang-like Syntax**

```erlang
send(ni', s, n, m) ->
    p = whereis\_name(ni', s, n),
    send(p, m).
```
Transitions of `whereis_name(ni',s,n)` and `whereis_name(s,n)` functions are presented above, and a transition of `send(p,m)` function is as follows:

\[
((grs,fgs,hhs,nds), send(p,m), ni) 
\rightarrow ((grs',fgs',hhs, nds'), p)
\]

where

\[
ni' \equiv \text{node}(p) \\
(fgs', nds') \equiv \text{AddConnections}(ni, ni', grs, fgs, hhs, nds)
\]

“node(p)” is a primitive distributed Erlang function `node(Pid) → Node` that identifies the node id of the node hosting process `p`. The passing of the message is not modelled in our state.

new_s_group. When we create a new s_group `s`, the s_group together with its nodes `nis` are added to the list of s_groups. If before joining the s_group nodes `nis` free then the nodes are removed from free groups. The new s_group has an empty namespace.

\[
((grs,fgs,hhs,nds), new\_s\_group(s,nis), ni) 
\rightarrow ((grs',fgs',hhs', nds''), (s, nis)) \quad \text{If } ni \in nis
\rightarrow ((grs,fgs,hhs,nds), \text{Error}) \quad \text{Otherwise}
\]

where

\[
nds' \equiv \text{InterConnectNodes}(nis, nds) \\
nds'' \equiv \text{AddSGroup}(s, nis, nds') \\
grs' \equiv grs \oplus \{(s, nis, \emptyset)\} \\
(fgs', hhs') \equiv \text{RemoveNodes}(nis, fgs, hhs)
\]

add_nodes. For node `ni` to add nodes identified by `nis` to s_group `s` node `ni` should belong to that s_group. S_group `s` membership is added to `nds'` nodes identified by `nis` node ids, and the existing and the new s_group nodes interconnect. The nodes that were free before joining the s_group are removed from the corresponding free groups `fgs'` and `hhs'`.

\[
((grs,fgs,hhs,nds), add\_nodes(s,nis), ni) 
\rightarrow ((grs'',fgs',hhs', nds''), (s, nis)) \quad \text{If } \text{IsSGroupSNode}(ni, s, grs)
\rightarrow ((grs,fgs,hhs,nds), \text{Error}) \quad \text{Otherwise}
\]

where

\[
\{(s, \{ni\} \oplus nis', ns)\} \oplus grs' \equiv grs \\
nds' \equiv \text{InterConnectNodes}(\{ni\} \oplus nis \oplus nis', nds) \\
nds'' \equiv \text{AddSGroup}(s, nis, nds') \\
grs'' \equiv \{(s, (\{ni\} \oplus nis) \oplus nis', ns)\} \oplus grs' \\
(fgs', hhs') \equiv \text{RemoveNodes}(nis, fgs, hhs)
\]

delete_s_group. To delete s_group `s` node defined by `ni` node id should be a member of that s_group. When s_group `s` is deleted its membership is removed from group names of `nds` identified by `ni` and
nis. In case the nodes become free, we add new free nodes to the corresponding free groups.

\[
\begin{align*}
&((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{delete}_\text{s\_group}(s), ni) \\
&\quad \rightarrow ((\text{grs}', \text{fgs}', \text{fhs}', \text{nds}''), \text{True}) & \text{If IsSGroupSNode}(ni, s, \text{grs}) \\
&\quad \rightarrow ((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{False}) & \text{Otherwise}
\end{align*}
\]

where

\[
\begin{align*}
\{(s, \{\{ni\} \oplus \text{nis}, ns\}) \oplus \text{grs}' \equiv \text{grs} \\
\text{nds}' \equiv \text{RemoveSGroup}(s, \{\{ni\} \oplus \text{nis}, ns) \\
(fgs', fhs', nds'') \equiv \text{NewFreeNodes}(fgs, fhs, nds')
\end{align*}
\]

**remove_nodes.** To remove nodes identified by nis node ids from s\_group s the nodes should be members of that s\_group. Additionally, ni node should be also a member of s\_group s and it cannot remove itself from an s\_group. The s\_group s membership is removed from group names of nds identified by nis. In case the nodes become free, we add new free nodes to the corresponding free groups.

\[
\begin{align*}
&((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{remove}_\text{nodes}(s, \text{nis}), ni) \\
&\quad \rightarrow ((\text{grs}'', \text{fgs}', \text{fhs}', \text{nds}''), \text{True}) & \text{If } ni \notin \text{nis} \land \text{IsSGroupSNode}(ni, s, \text{grs}) \\
&\quad \rightarrow ((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{False}) & \text{Otherwise}
\end{align*}
\]

where

\[
\begin{align*}
\{(s, \{\{ni\} \oplus \text{nis}, ns\}) \oplus \text{grs}' \equiv \text{grs} \\
\text{grs}'' \equiv \{(s, \{\{ni\} \oplus \text{nis}', ns\}) \oplus \text{grs}' \\
\text{nds}' \equiv \text{RemoveSGroup}(s, \text{nis}, \text{nds}) \\
(fgs', fhs', nds'') \equiv \text{NewFreeNodes}(fgs, fhs, nds')
\end{align*}
\]

**own_nodes** function has two versions, i.e. own\_nodes/0 and own\_nodes/1. When no parameter is specified, i.e. own\_nodes(), the function returns a list of nis' node ids of the nodes that belong to the same groups as node ni.

\[
\begin{align*}
&((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{own}_\text{nodes}(), ni) \\
&\quad \rightarrow ((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{nis}')
\end{align*}
\]

where

\[
\text{nis}' \equiv \text{OwnGroupNis}(ni, \text{grs}, \text{fgs}, \text{fhs})
\]

When s\_group s is specified, i.e. own\_nodes(s) the function returns a list of nis' node ids of the nodes that belong to the s\_group. In case node ni does not belong to s\_group s, an empty list is returned.

\[
\begin{align*}
&((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{own}_\text{nodes}(s), ni) \\
&\quad \rightarrow ((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \text{nis}') & \text{If IsSGroupSNode}(ni, s, \text{grs}) \\
&\quad \rightarrow ((\text{grs}, \text{fgs}, \text{fhs}, \text{nds}), \{\}) & \text{Otherwise}
\end{align*}
\]

where

\[
\begin{align*}
\{(s, \{\{ni\} \oplus \text{nis}, ns\}) \oplus \text{grs}' \equiv \text{grs} \\
\text{nis}' \equiv \{\{ni\} \oplus \text{nis}
\end{align*}
\]
own_s_groups function returns a list of s_groups and their nodes to which \( ni \) node belongs to if node \( ni \) is an s_group node, empty list otherwise.

\[
\begin{aligned}
((grs,fgs,fhs,nds), \text{own_s_groups}(\cdot), ni) \\
\quad \rightarrow ((grs,fgs,fhs,nds), \text{snis}) & \quad \text{If IsSGroupNode(ni, grs)} \\
\quad \rightarrow ((grs,fgs,fhs,nds), \emptyset) & \quad \text{Otherwise}
\end{aligned}
\]

where

\[ \text{snis} \equiv \text{OutputOwnSGroups}(ni, grs) \]

### 5.2.2 Auxiliary Functions

In this section we present auxiliary functions introduced in Section 5.2.1.

**IsSGroupNode** function returns True if node \( ni \) is a member of some s_group \( s \), False otherwise.

\[
\text{IsSGroupNode}(ni, grs) = \exists s, nis, ns, grs' . \{(s, \{ni\} \oplus nis, ns)\} \oplus grs' \equiv grs
\]

**IsFreeNormalNode** function returns True if node \( ni \) is a member of some free normal group, False otherwise.

\[
\text{IsFreeNormalNode}(ni, fgs) = \exists nis, ns, fgs' . \{(\{ni\} \oplus nis, ns)\} \oplus fgs' \equiv fgs
\]

**IsFreeHiddenNode** function returns True if node \( ni \) is a member of some free hidden group, False otherwise.

\[
\text{IsFreeHiddenNode}(ni, fhs) = \exists ns, fhs' . \{(ni, ns)\} \oplus fhs' \equiv fhs
\]

**SameFreeGroup** function returns True if nodes \( ni \) and \( ni' \) are members of the same free normal group, False otherwise.

\[
\text{SameFreeGroup}(ni, ni', fgs) = \exists nis, ns, fgs' . \{(\{ni, ni'\} \oplus nis, ns)\} \oplus fgs' \equiv fgs
\]

**SameSGroup** function returns True if nodes \( ni \) and \( ni' \) are members of the same \( s \_\text{group} \), False otherwise.

\[
\text{SameSGroup}(ni, ni', grs) = \exists s, nis, ns, grs' . \{(s, \{ni, ni'\} \oplus nis, ns)\} \oplus grs' \equiv grs
\]

**AddConnections** function systematically checks types of nodes \( ni \) and \( ni' \), and then adds corresponding connections. Here, a connection between the nodes may already exist. The first outcome \((fgs, nds)\) occurs when the nodes are in the same free normal group or the same \( s \_\text{group} \) and hence are connected. The second outcome \((fgs, nds')\) occurs in one of the following cases: 1) node \( ni \) is an \( s \_\text{group} \) node, and nodes \( ni \) and \( ni' \) are not in the same \( s \_\text{group} \); note that \( ni' \) may be an \( s \_\text{group} \) node, a free normal node, or a free hidden node; 2) node \( ni \) is a free hidden node; 3) node \( ni \) is a free normal node and node \( ni' \) is either an \( s \_\text{group} \) node or a free hidden node. The third outcome \((fgs'', nds'')\)
occurs when nodes $ni$ and $ni'$ are in different free normal groups.

$$\text{AddConnections}(ni, ni', grs, fgs, fhs, nds) = (fgs, nds)$$
If $\text{SameFreeGroup}(ni, ni', fgs)$
$\lor \text{SameSGroup}(ni, ni', grs)$

$$= (fgs, nds')$$
If $(\text{IsSGroupNode}(ni, grs) \land \neg \text{SameSGroup}(ni, ni', grs))$
$\lor \text{IsFreeHiddenNode}(ni, fhs)$
$\lor (\text{IsFreeNormalNode}(ni, fgs) \land \neg \text{IsFreeNormalNode}(ni', fgs))$

$$= (fgs'', nds'')$$
If $ni \in nis1 \land ni' \in nis2$

$= (fgs, nds)$
Otherwise

where

$$nds' \equiv \text{InterConnectNodes}\{\{ni, ni'\}, nds\}$$
$\{(\{nис1, nис1\}) \oplus \{(nис2, nис2)\}) \oplus fgs' \equiv fgs$

$$fgs'' \equiv \{(nис1 \oplus nис2, nис1 \oplus nис2)\} \oplus fgs'$$
$$nds'' \equiv \text{InterConnectNodes}(nис1 \oplus nис2, nds)$$

$\text{InterConnectNodes}$ function interconnects nodes from $nds$ identified by $nis$ node ids.

$\text{InterConnectNodes}(nis, nds) = nds \cup \{(ni, nt, (cs \oplus nис) - \{ni\}, gs) \mid (ni, nt, cs, gs) \in nds, ni \in nис\}$

$\text{AddSGroup}$ function adds membership of s_group $s$ to all nodes identified by $nis$ node ids.

$\text{AddSGroup}(s, nis, nds) = (nds - nds') \oplus nds''$

where

$$nds' \equiv \{(ni, nt, cs, gs) \mid (ni, nt, cs, gs) \in nds, ni \in nис\}$$

$$nds'' \equiv \{(ni, nt, cs, AddSGroupS(s, gs)) \mid (ni, nt, cs, gs) \in nds'\}$$

$\text{AddSGroupS}$ function adds $s$ group $s$ to a list of s_groups.

$\text{AddSGroupS}(s, gs)$

$$= \{s\}$$
If $gs \equiv \text{NoGroup}$

$$= gs \oplus \{s\}$$
Otherwise

$\text{RemoveSGroup}$ function removes membership of s_group $s$ from all nodes identified by $nis$ node ids.

$\text{RemoveSGroup}(s, nis, nds) = (nds - nds') \oplus nds''$

where

$$nds' \equiv \{(ni, nt, cs, gs) \mid (ni, nt, cs, gs) \in nds, ni \in nис\}$$

$$nds'' \equiv \{(ni, nt, cs, gs) \mid (ni, nt, cs, gs \oplus \{s\}) \in nds'\}$$
RemoveNodes function removes node ids identified by \( nis \) from free normal groups \( fgs \) and free hidden groups \( fhs \).

\[
\text{RemoveNodes}(nis, fgs, fhs) = (fgs'', fhs')
\]

where

\[
fhs' = fhs - \{(ni, ns) | (ni, ns) \in fhs, ni \in nis\}
\]

FindName function identifies pid \( p \) of name \( n \) registered in group \( s \). Here, \( s \) can be \text{NoGroup} or an s\_group name.

\[
\text{FindName}(ni, s, n, grs, fgs, fhs)
\]

\[
\begin{align*}
= p1 & \quad \text{If } s \equiv \text{NoGroup} \land nt \equiv \text{Normal} \\
= p2 & \quad \text{If } s \equiv \text{NoGroup} \land nt \equiv \text{Hidden} \\
= p3 & \quad \text{Otherwise}
\end{align*}
\]

where

\[
\begin{align*}
\{(ni, nt, cs, gs)\} \oplus nds' & \equiv nds \\
\{(ni) \oplus nis, \{(n, p1)\} \oplus ns1\} \oplus fgs' & \equiv fgs \\
\{(ni, \{(n, p2)\} \oplus ns2\}) \oplus fhs' & \equiv fhs \\
\{(s, \{ni\} \oplus nis, \{(n, p3)\} \oplus ns3)\} \oplus grs' & \equiv grs \\
\end{align*}
\]

NewFreeNodes function places new free nodes to corresponding free groups. The groups are determined by the node types and connections.

\[
\text{NewFreeNodes}(fgs, fhs, nds) = (fgs', fhs', nds'')
\]

where

\[
\begin{align*}
nds' & \equiv \{(ni, nt, cs, \text{NoGroup}) | (ni, nt, cs, \{\}) \in nds\} \\
fhs' & \equiv \text{AddFreeHiddenGroup}(fhs, nds') \\
(fgs', nds'') & \equiv \text{AddFreeNormalGroup}(fgs, nds, nds') \\
\end{align*}
\]

AddFreeHiddenGroup function adds new free hidden groups to \( fhs \).

\[
\text{AddFreeHiddenGroup}(fhs, nds) = fhs \oplus \{(ni, \{\}) | (ni, \text{Hidden}, cs, \text{NoGroup}) \in nds\}
\]

AddFreeNormalGroup function distributes new free normal nodes defined by \( nis \) from \( nds' \) to corresponding free normal groups. The groups are defined by the node connections \( css' \) to existing free nodes. In case a new node has no connections with existing free normal nodes a new free normal group is created.

\[
\text{AddFreeNormalGroup}(fgs, nds, nds') = (fgs', nds'')
\]

where

\[
\begin{align*}
nis & \equiv \{ni | (ni, \text{Normal}, cs, \text{NoGroup}) \in nds'\} \\
\text{css} & \equiv \bigoplus \{cs | (ni, \text{Normal}, cs, \text{NoGroup}) \in nds'\} \\
\text{css'} & \equiv \{ni | (nis' \oplus \{ni\}, ns) \in fgs, ni \in css\} \\
(fgs', nds'') & \equiv \text{MergeFreeNormalGroups}(nis, css', fgs, nds)
\end{align*}
\]
MergeFreeNormalGroups function merges free normal groups which nodes are connected with each other. The function also adds missing connections between nodes from the same free normal group.

\[
\text{MergeFreeNormalGroups}(\text{nls}, \text{css}, \text{fgs}, \text{nds}) = (\text{fgs}, \text{nds}) \quad \text{If } \text{nls} \equiv \{\}\n\]
\[
= (\text{fgs}', \text{nds}) \quad \text{If } \text{nls} \neq \{\} \land \text{css} \equiv \{\}\n\]
\[
= (\text{fgs}'', \text{nds}') \quad \text{Otherwise}\n\]

where
\[
\text{fgs}' \equiv \text{fgs} \oplus \{(\text{nls}, \{\})\}\n\]
\[
\text{fgs},r \equiv \text{FindFreeNormalGroups}(\text{css}, \text{fgs})\n\]
\[
\text{nls}1 \equiv \bigoplus\{\text{nls}' \mid (\text{nls}', \text{ns}) \in \text{fgs},r\}\n\]
\[
\text{nls}'' \equiv \text{nls}1 \oplus \text{nls}\n\]
\[
\text{ns}' \equiv \bigoplus\{\text{ns} \mid (\text{nls}', \text{ns}) \in \text{fgs},r\}\n\]
\[
\text{fgs}'' \equiv (\text{fgs} - \text{fgs},r) \oplus \{(\text{nls}'', \text{ns}')\}\n\]
\[
\text{nds}' \equiv \text{InterConnectNodes}(\text{nls}'', \text{nds})\n\]

FindFreeNormalGroups function returns a list of free normal groups that contain nodes identified by nls node ids.

\[
\text{FindFreeNormalGroups}(\text{nls}, \text{fgs}) = \{\{\text{nls}' \oplus \{\text{ni}\}, \text{ns}\} \mid (\text{nls}' \oplus \{\text{ni}\}, \text{ns}) \in \text{fgs}, \text{ni} \in \text{nls}\}\n\]

OwnGroupNis function returns a list of nls' node ids of the nodes that belong to the same groups as ni node.

\[
\text{OwnGroupNis}(\text{ni}, \text{grs}, \text{fgs}, \text{fhs}) = \{\{\text{ni} \oplus \{\text{ni}\}, \text{ns}\} \mid (\text{ni} \oplus \{\text{ni}\}, \text{ns}) \in \text{fgs}, \text{ni} \in \text{nls}\}\n\]

OwnSGroupNis function returns a list of nls' node ids of the nodes that belong to the same s_groups as ni node.

\[
\text{OwnSGroupNis}(\text{ni}, \text{grs}) = \text{nls}' \oplus \{\text{ni}\}\n\]

where
\[
\{\{\text{ni} \oplus \text{nls}, \text{ns}\}\} \oplus \text{fgs}' \equiv \text{fgs}\n\]
\[
\text{nls}' \equiv \text{OwnSGroupNis}(\text{ni}, \text{grs})\n\]

OutputNs function returns a list of process names registered in ns namespace of s_group s.

\[
\text{OutputNs}(\text{s}, \text{ns}) = \{(\text{s}, \text{n}) \mid (\text{n}, \text{p}) \in \text{ns}\}\n\]
**OutputOwnSGroups** function returns information about s_groups to which *ni* node belongs to, i.e. a list of s_group names *s* and node ids of nodes from those s_groups.

\[
\text{OutputOwnSGroups}(ni, grs) = \{(s, \{ni\} \oplus nis) \mid (s, \{ni\} \oplus nis, ns) \in grs\}
\]

# 6 Validation of S_group Semantics and Implementation

In addition to defining the operational semantics of SD Erlang, we have also validated the consistency between the formal semantics and the SD Erlang implementation. Our approach is based on *property-based testing*, in particular we use the Erlang QuickCheck [AHJW06] tool developed by Quivq. The testing revealed s_group bugs in both the semantics and the s_group implementation; it gives us high confidence on the consistency between the formal semantics of s_groups and the implementation. Although SD Erlang is the subject under test in this case, the testing approach is applicable to the validation of formal operational semantics in general. While Quivq QuickCheck is the tool we use for testing, another property-based testing tool *PropEr* [PAS11] written in Erlang and inspired by QuickCheck could be an alternative.

In this section we provide a brief introduction of property-based testing and QuickCheck’s *eqc_statem* in Section 6.1, and an overview of the testing approach in Section 6.2.

## 6.1 Property-based Testing and QuickCheck

Property-Based Testing (PBT) provides a powerful and high-level approach to testing. In PBT system behaviour is specified by properties expressed in a logical form rather than focusing on individual test cases. For example, a function without side effects may be specified by means of the full input/output relation using a universal quantification over all the inputs; a stateful system will be described by means of a model, which is an extended finite state machine. The system is then tested by checking whether it has the required properties for randomly generated data; the randomly generated data may be inputs to functions, sequences of API calls to the stateful system, or other representations of test cases.

QuickCheck supports random testing of Erlang programs. Properties of the programs are stated in a subset of first-order logic embedded in Erlang syntax. QuickCheck verifies these properties for collections of randomly generated Erlang values with user guidance in defining the generators where necessary. When a counterexample is found, QuickCheck tries to generate a simpler – and thus more comprehensible – counterexample, in a constructive manner; this process is called *shrinking*. *PropEr* testing tool has a very similar user interface.

QuickCheck is the tool of choice for the testing of SD Erlang developed by QuivQ. In particular, we utilise its support for extended finite state machines through the *eqc_statem* behaviour. As illustrated

![QuickCheck eqc_statem](image)

**Figure 4: QuickCheck eqc_statem**
in Figure 4, with \texttt{eqc_statem} the user defines an abstract model of the System Under Test (SUT), including an abstraction of the state of the SUT itself. The model includes an initial abstract state in which test cases begin and describes how each command changes the state. The state is used by QuickCheck both during test case generation and during test execution. For each command a number of properties can be described, e.g.

- \textit{preconditions} to decide whether or not to include a candidate command in test cases;
- \textit{postconditions} to check whether the value returned by the executed command is correct;
- a description of the changes in the abstract state as a result of command execution;
- a method to generate an appropriate function call to appear next in a test case.

\section*{6.2 Validation of SD Erlang}

To validate the SD Erlang semantics and implementation using \texttt{eqc_statem}, an abstract model of the SUT is needed as discussed in Section 6.1. Our approach, as illustrated in Figure 5, is to derive the abstract model from the semantics specification, i.e. we translate the semantics specification into an abstract model written in Erlang. The abstract model contains the abstract state representation and the transition from one state to another when an operation is applied. There is a transition function for each SD Erlang operation defined in the semantics.

An Erlang representation of the abstract state (Figure 6) shows a clear mapping from the formal state specification to the Erlang representation. The abstract model state is initialised according to the actual distributed SD Erlang system that we test. Thus, both the abstract state machine and the tested system have the same starting point. For our actual testing setup we use 14 SD Erlang nodes, 12 of which are normal nodes and 2 are hidden nodes.

A collection of data generators have been defined to guide the automatic generation of data value fed to \texttt{s_group} operations; the callback function \texttt{command/1} allows QuickCheck to automatically generate candidate test commands (Figure 7). The arguments supplied to each symbolic call of an \texttt{s_group} operation are also data generators. The \texttt{frequency/1} function makes a weighted choice between the generators in its argument, such that the probability of choosing each generator is proportional to the weight paired with it. A test case is a sequence of \texttt{s_group} operations, and by default, QuickCheck generates 100 test cases for each run unless the testing fails.

The conditions defined in \texttt{precondition/2} function serve as a filter, so a candidate command is included in a test case only if the preconditions for that operation are met. For instance, the precondition for \texttt{new_s_group} operation is defined as follows:

![Figure 5: Testing SD Erlang Using QuickCheck eqc_statem](image)
-record(state, {groups =[], free_groups =[], free_hidden_groups =[], nodes =[]}).

-type group()::{s_group_name(), [node_id()], namespace()}.
-type free_group()::{[node_id()], namespace()}.
-type free_hidden_group()::{node_id(), namespace()}.
-type a_node()::{node_id(), node_type(), connections(), gr_names()}.

-type gr_names()::free_normal_group|free_hidden_group|[s_group_name()].
-type namespace()::[{atom(), pid()}].
-type connections()::[node_id()].
-type node_type()::visible|hidden.
-type s_group_name()::atom().
-type node_id()::node().

Figure 6: An Abstract State Representation in Erlang

precondition(_S, {call, ?MODULE, new_s_group, 
[[_SGroupName, NodeIds, _CurNode], 
_AllNodeIds]) ->
NodeIds/=[];  % A new s_group cannot be empty

Each generated test command is applied to both the abstract model and the actual tested system. The application of the test command to the abstract model takes the abstract model from its current state to a new state as described by the transition functions; whereas the application of the test command to the real system leads the system to a new actual state. The actual s_group-related state information is collected from each tested node, then the information is merged and normalised to the same format as the abstract state representation. For a successful testing, after the execution of each test command the normalised actual state should be the same as the abstract state; this constraint, together with some other generic invariants, is specified as the postconditions of s_group operations. For instance, the postcondition for new_s_group operation is defined as follows:

-- Res is the actual value returned by the command
-- ActualState is the actual state information collected from nodes
postcondition(S, {call, ?MODULE, new_s_group, 
[[_SGroupName, NodeIds, CurNode], 
_AllNodeIds]),
(Res, ActualState)) ->
(AbsRes, NewS) = new_s_group_next_state(S, _SGroupName, NodeIds, CurNode),
(AbsRes == Res) and is_the_same(ActualState, NewS);

Postconditions are validated after execution of every test command. If a validation fails the failing test case is reported.

The top-level property that connects all the parts together is defined as follows:

prop_s_group() ->
?SETUP(
 fun setup/0, 
 ?FORALL(Cmds, commands(?MODULE),
 begin
 (H,S,Res) = run_commands(?MODULE,Cmds),
 teardown(),
 setup(),
 pretty_commands(?MODULE, Cmds, {H,S,Res}, Res==ok)
 end)).
command(S) ->
    frequency(
        [[5, {call, ?MODULE, new_s_group,
                [gen_new_s_group_pars(S), all_node_ids(S).Lib]]}]
        ,[5, {call, ?MODULE, add_nodes,
                [gen_add_nodes_pars(S), all_node_ids(S).Lib]])
        ,[5, {call, ?MODULE, remove_nodes,
                [gen_remove_nodes_pars(S), all_node_ids(S).Lib]])
        ,[5, {call, ?MODULE, delete_s_group,
                [gen_delete_s_group_pars(S), all_node_ids(S).Lib]])
        ,[10, {call, ?MODULE, register_name,
                [gen_register_name_pars(S), all_node_ids(S).Lib]}]
        ,[10, {call, ?MODULE, whereis_name,
                [gen_whereis_name_pars(S), all_node_ids(S).Lib]}]
        ,[10, {call, ?MODULE, re_register_name,
                [gen_re_register_name_pars(S), all_node_ids(S).Lib]}]
        ,[10, {call, ?MODULE, unregister_name,
                [gen_unregister_name_pars(S), all_node_ids(S).Lib]}]
        ,[10, {call, ?MODULE, send,
                [gen_send_pars(S), all_node_ids(S).Lib]}]
        ,[1, {call, ?MODULE, node_down_up,
                [gen_node_down_up_pars(S), all_node_ids(S).Lib]}]
    ).

Figure 7: Command Generator

Here, setup/0 and teardown/0 functions start and terminate the actual SD Erlang system respectively.

To date we have validated the nine functions listed in Figure 7. We found and fixed four bugs in the semantics and identified an issue in the SD Erlang implementation, i.e. in some cases after an s_group has been removed the actual s_group structure and the abstract s_group structure initially do not match each other but eventually become the same. The structures become identical after a 60s timeout and an error report from the real system. Currently, we are not sure about the origins of the issue but we believe it is due to node synchronisation, and work on resolving of the issue.

7 Implications and Future Work

This deliverable addresses SD Erlang s_group reliability and s_group semantics. The reliability was addressed by implementing a node recovery mechanism and analysing s_groups that have the same name. To identify the most suitable node recovery mechanism we explored several alternate strategies using system state estimation and analysing RELEASE SD Erlang exemplars (Section 3). We outline the approach we have implemented where each node keeps its own s_group information in a configuration file that is updated by the node every time the s_group configuration changes. To guarantee a high probability of s_group name uniqueness when s_groups are started dynamically we have modified function s_group:new_s_group to generate names that are very probably unique (Section 4). If duplicate s_group names do occur we have ensured correct behaviour of s_groups and nodes. We have also introduced s_group transition semantics by defining an abstract state of SD Erlang systems and presented the transitions of fifteen SD Erlang functions (Section 5). We have validated the consistency between the formal semantics and the SD Erlang implementation using Erlang QuickCheck testing tool.
Our future plans include the following.

- **SD Erlang Semantics.** Together with the Kent team continue to work on the semantics by running more QuickCheck tests, relaxing assumptions, and the outstanding SD Erlang implementation issue.

- **SD Erlang Validation.** Validating SD Erlang by running the benchmarks discussed in Section 3.2, and releasing the SD Erlang library in deliverable D3.4: Scalable Reliable OTP Library Release.

- **Semi-explicit Placement.** Conduct the work outlined in deliverable D3.2: Scalable SD Erlang Computation Model [REL13a, Section 6].

### Change Log

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</tr>
<tr>
<td>0.2</td>
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<td>Revised version based on comments from N. Papaspyrou submitted to the Commission Services</td>
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### References


